CSCI-2500 Computer Organization
Carry-Lookahead (CLA) Adder

Justin M. LaPre
Department of Computer Science
Rensselaer Polytechnic Institute
110 8th Street
Troy, New York U.S.A. 12180

March 14, 2016

1 Equation Dependencies for 64 Bit CLA

Recall that we can express the sum of two numbers as:

$$\text{sum}_i = a_i \bigoplus b_i \bigoplus c_{i-1} \tag{1}$$

Also, we know that:

$$c_i = g_i + p_i c_{i-1} \tag{2}$$

where g_i is the *generate function* which says did we generate a carry in the i^{th} stage and the p_i is the *propagate function* which says did we propagate a carry in the i^{th} stage assuming the carry-in, c_{i-1}, was positive. This yields the following:

$$g_i = a_i \times b_i \tag{3}$$

$$p_i = a_i + b_i \tag{4}$$

$$c_i = g_i + p_i c_{i-1} \tag{5}$$

Now, using the above recurrence we can find what c_i is for any 4 bit block or *group*.

$$c_i = g_i + p_i c_{i-1} \tag{6}$$
$$c_{i+1} = g_{i+1} + p_{i+1} c_i \tag{7}$$
$$c_{i+2} = g_{i+2} + p_{i+2} c_{i+1} \tag{8}$$
$$c_{i+3} = g_{i+3} + p_{i+3} c_{i+2} \tag{9}$$

Notice how each of the c_i equations can all be written in terms of the the g, p and c_{i-1}. But, c_{i-1} is really the carry-in for this *group* of 4 bits. So, this means that the carry-in to those groups depends on the gc equations, which are:

$$gc_j = gg_j + gp_j gc_{j-1} \tag{10}$$
$$gc_{j+1} = gg_{j+1} + gp_{j+1} gc_j \tag{11}$$
$$gc_{j+2} = gg_{j+2} + gp_{j+2} gc_{j+1} \tag{12}$$
$$gc_{j+3} = gg_{j+3} + gp_{j+3} gc_{j+2} \tag{13}$$
$$\tag{14}$$

where...

$$gg_j = g_{i+3} + p_{i+3} g_{i+2} + p_{i+3} p_{i+2} g_{i+1} + p_{i+3} p_{i+2} p_{i+1} g_i \tag{15}$$
$$gp_j = p_{i+3} p_{i+2} p_{i+1} p_i \tag{16}$$

Again, notice how each of the gc_j equations can all be written in terms of the the gg, gp and gc_{j-1}. But, gc_{j-1} is really the carry-in for this *section* of 4 groups. So, this means that the carry-in to those sections depends on the sc equations, which are:

$$sc_k = sg_k + sp_k sc_{k-1} \tag{17}$$
$$sc_{k+1} = sg_{k+1} + sp_{k+1} sc_k \tag{18}$$
$$sc_{k+2} = sg_{k+2} + sp_{k+2} sc_{k+1} \tag{19}$$
$$sc_{k+3} = sg_{k+3} + sp_{k+3} sc_{k+2} \tag{20}$$
$$\tag{21}$$

where...

$$sg_k = gg_{j+3} + gp_{j+3} gg_{j+2} + gp_{j+3} gp_{j+2} gg_{j+1} + gp_{j+3} gp_{j+2} gp_{j+1} gg_j \tag{22}$$
$$sp_k = gp_{j+3} gp_{j+2} gp_{j+1} gp_j \tag{23}$$

2 Steps for Calculation for 64 Bit CLA

1. Calculate g_i and p_i for all i. (1 gate delay)
2. Calculate gg_j and gp_j for all j using g_i and p_i. (2 gate delays)
3. Calculate sg_k and sp_k for all k using gg_j and gp_j. (2 gate delays) *Note, it is at this point, we can shift to computing the top-level sectional carries. This is because the number of sections is less than or equal the block size which is 4 bits.*
4. Calculate sc_k using sg_k and sp_k for all k and 0 for sc_{i-1}. (2 gate delays)
5. Calculate gc_j using gg_j, gp_j and correct $sc_k, k = (j \text{ div } 4)$ as sectional carry-in for all j. (2 gate delays)
6. Calculate c_i using g_i, p_i and correct $gc_j, j = (i \text{ div } 4)$ as group carry-in for all i. (2 gate delays)
7. Calculate sum$_i$ using $a_i \bigoplus b_i \bigoplus c_i$ for all i. (2 gate delays)
8. **Total gate delays for 64 bit CLA is 13.**